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Prepared by: Wendy Calise
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Probability

Children already have informal experience with the idea of probability. But they should start their work with some exploration to bring that knowledge to their consciousness.

Materials: Several dice (with different numbers of sides and even one that has 1 on two sides, 2 on two sides, and 3 on the other two sides; 15 blue, 10 red and 5 white poker chips; bag; deck of playing cards; labels)

Introduction to Probability

Today we are going to talk a bit about probability. It is something that you are already familiar with, but you might not recognize the name.

Look at the poker chips that we have here.

If I put them all in the bag, which do you think that I would be most likely to pick? (Blue)

Why do you think that I would pick blue? (Because there are more blue than any other color). *Does that mean that I will definitely pick blue?* (No)

What chip do you think I would be least likely to pick? (white)

Why? (Because there are fewest of the white chips) *Does that mean that I definitely will not get white?* (No) *But I would be least likely to pick a white.*

This idea of likelihood is called probability. Probability is the likelihood that some outcome will occur. We decided that the chip that would most likely be picked was the blue chip. We can also say that picking a blue chip has the highest probability. What chip has the lowest probability of being picked? (white)

We have several different things on this tray that allow you to explore probability. Take this work out later and see how often the most likely outcome occurs. For example, if you consider the deck of cards, would you be more likely to pick a number card or a face card? (number card because there are more of those). *Now I want to actually choose a card. What did you get?* (a face card – you don't always get what has the higher probability) *So that is all I am asking you to do. You can work with the dice, the cards and the chips. See how long it takes you to get something that has the least probability.*

We will meet in a few days to see what you found from your exploration and to learn a bit more.

Theoretical and Experimental Probability and Mathematical Expression

Gather the children again and just ask them to share their experiences. The purpose of the exploration is just to solidify what they already know from experience but also to bring to their attention that probability is not a guarantee.

Today I want to show you a way to express probability mathematically.

How many chips are there in the bag? (30)
How many blue? (15)
How many red? (10)
How many white? (5)

And which has the highest probability of being chosen? (Blue)

When I reach into the bag and take out a chip, the result is called an outcome.

Pick a chip out of the bag. I want you to get a blue chip. You got a blue chip. We call that a favorable outcome because that is the chip that I wanted. Put the blue chip back in the bag.

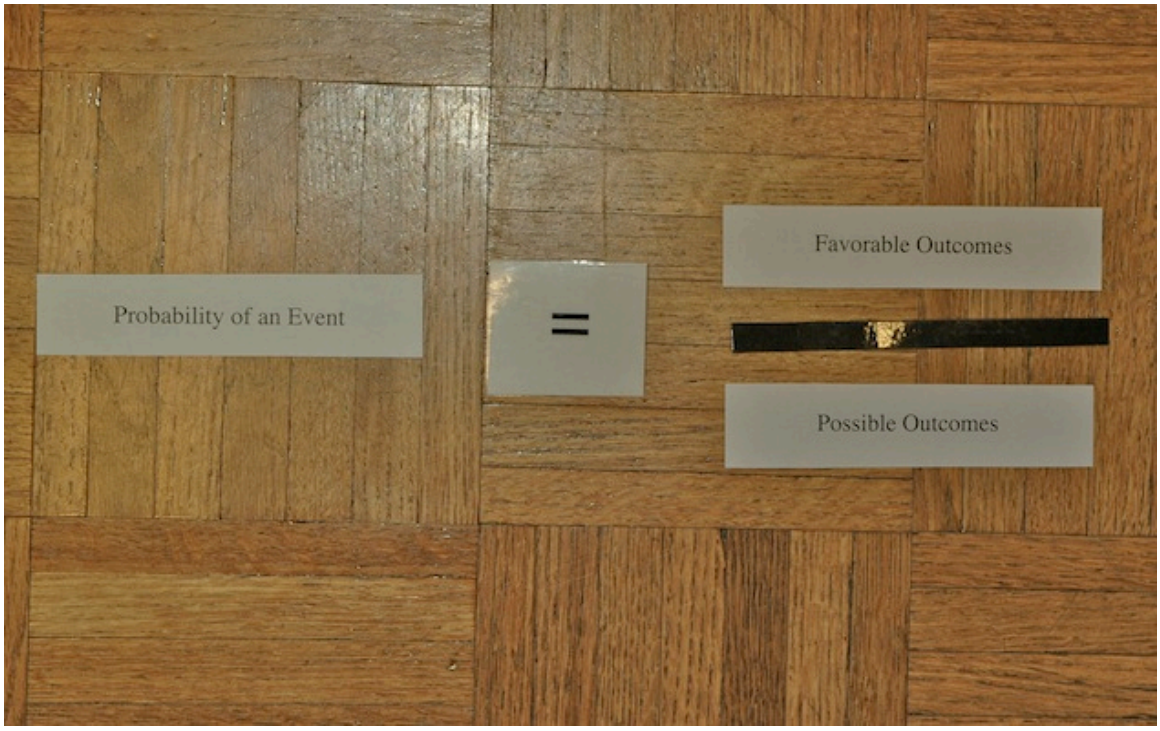
Pick another chip.

Have children pick until they get a red or a white, always returning the chip to the bag before choosing another.

Now you got a white chip. But I wanted blue. We call this outcome unfavorable. If all of the chips are in the bag, how many possible outcomes are there? (If they don't know, ask them how many chips are in the bag). *There are 30 possible outcomes because there are 30 chips and any one of those chips can be picked.*

So, what if I wanted to get a red chip? This is how I could write the probability down.

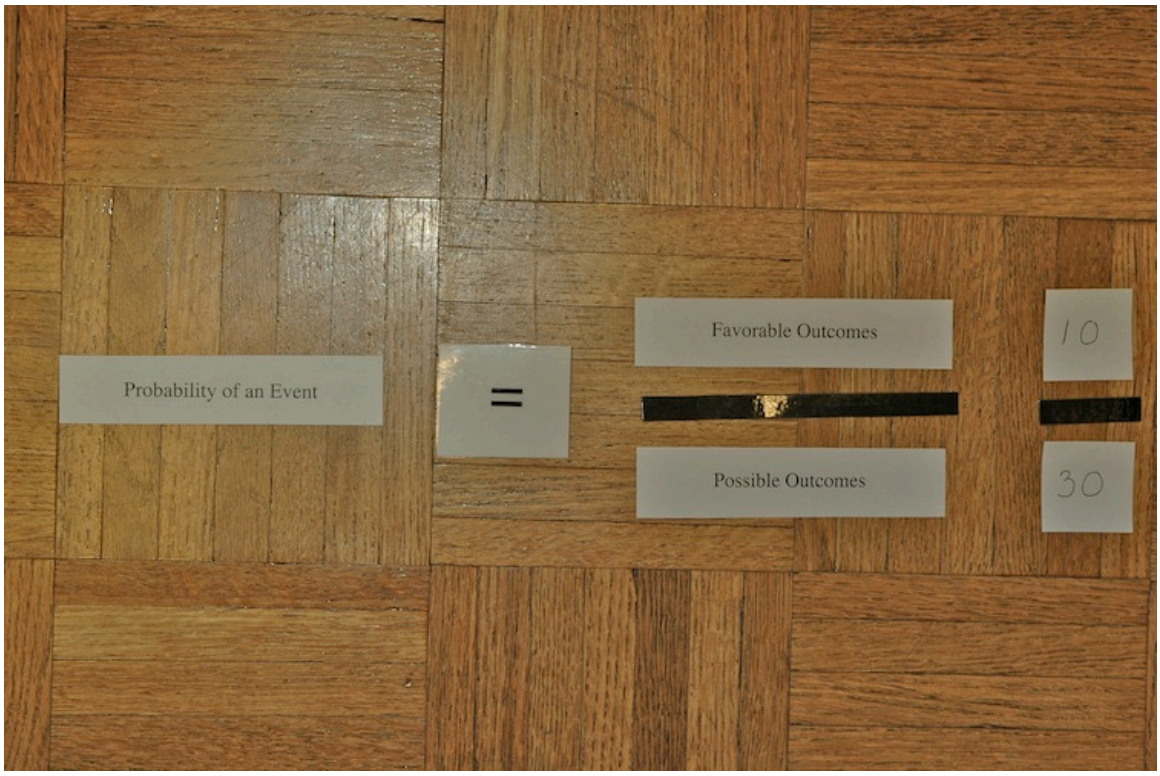
(Arrange the labels, probability of an event, possible outcomes, favorable outcomes, and black strip).



How many chips in the bag would give me a favorable outcome? (10) Write ten on a square. *How many chips are in the bag altogether?* (30) *So how many possible outcomes are there?* (30) Write 30 on another slip.

Probability can be expressed in the form of a fraction. The number on top is the number of favorable outcomes. Write the number ten on a paper square and place it above the small black strip.

The number below is the number of possible outcomes. Place 30 below the strip.



We can say that the probability of picking a red chip is ten out of thirty. And what can we do to this fraction? (Reduce it.) *And that would make it one out of three. So the probability of choosing a red chip is 1 out of 3.*

That means that if we pick from the bag three times, we would expect to get a red chip how many times? (One)

Will that always happen? Will we definitely get a red chip every three times we choose? Try it now. Prove that it will not happen that way by repeating this several times.

That is because when we actually do it, we get something called experimental probability. Experimental probability is much like a science experiment. Sometime we get what we expect, sometimes we don't.

But the probability that we determined to be 1 out of 3 is called the theoretical probability. That means that in theory, this is what I expect to happen. But in practice it might not always happen that way.

So next I would like you to explore again. This time I want you to decide what you want to happen, your favorable outcome. Let's say choosing a white chip. Write the following in your journal:

Favorable outcome: white chips (5)
Possible outcomes: all chips (30)

Then write it as a fraction and reduce.

5/30 or 1/6.

Now write:

Theoretical Probability: 1/6

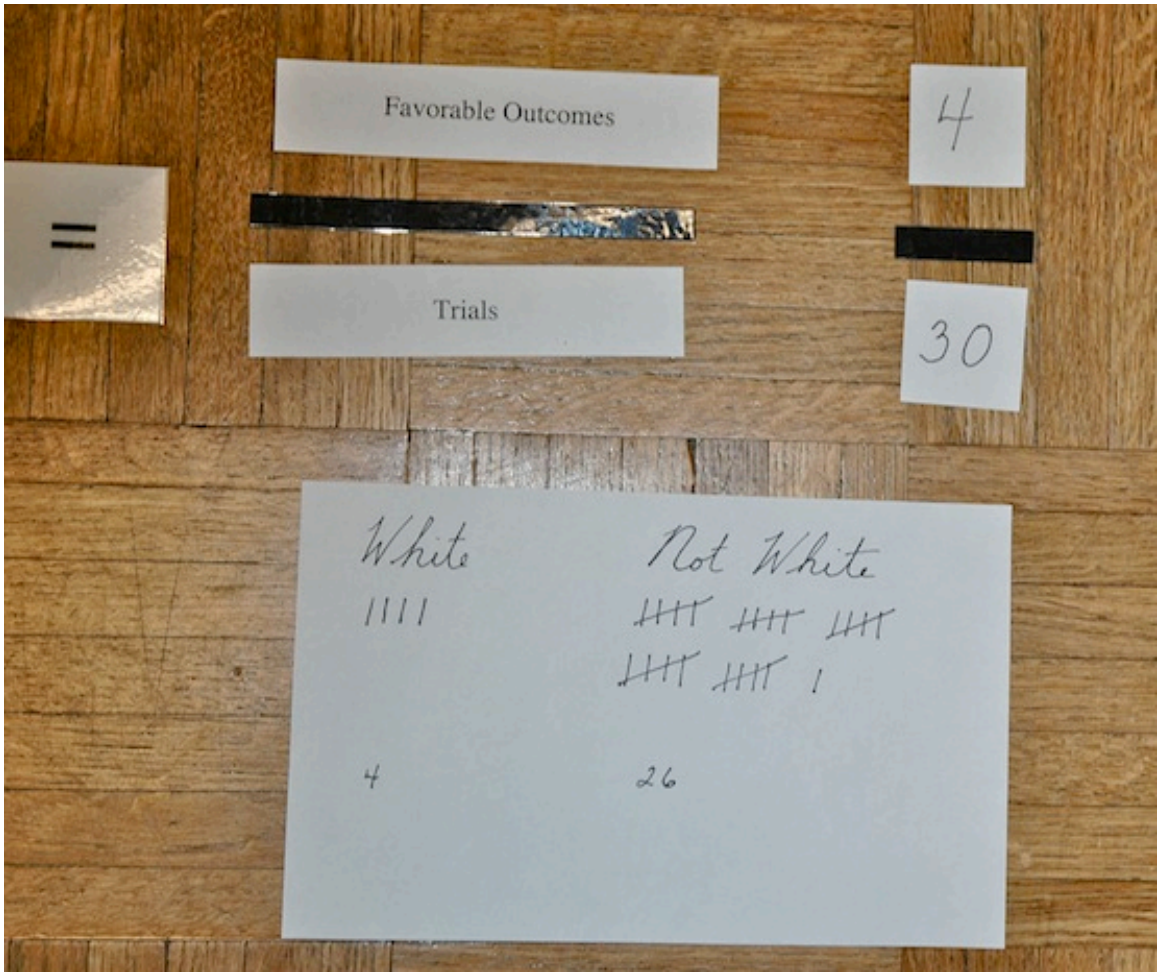
Next I want to you experiment. In this case, the number above the line is still the favorable outcomes, but the number below is the number of trials, which is how many times you tried.

So you can actually try as many times as you want. Not just 6 times. And I would like you to try more than that in some cases and record your answers.

Let's try it now.

Have the children make a series of trials making hash marks to keep track of how many times they get white (favorable) and how many times they get not white (unfavorable). When finished, count the sum of hash marks in **both** columns. This is the number of trials which goes under the fraction line.

Arrange the labels: Favorable outcomes above black line, Trials below. Match the corresponding number tickets that you write.



We will meet again in a few days, and you can show me what you found.

When you meet again you want to notice with the children that the more trials they attempted, the closer to the theoretical probability they got. For example: if they want to get a white chip and they tried it, they might get 2/6 or 0/6 in six trials. But the more trials they conduct, the closer they will get to 1/6, which is the theoretical probability. You will make the comparison by writing the favorable outcomes over trials as a fraction and then comparing to the theoretical fraction.

Complementary Events

Choose one of the dice and roll it. Roll it again. What is the probability of rolling a three? 1/6. What is the probability that you will not roll a three? 5/6 because there are five other numbers that could come up that would not

be a three. The probability of an event occurring and the probability that the event will not occur are called complementary events.

So in this case we have 1/6 and 5/6.

Let's try another, drawing a red chip from our bag:

Probability that you will pick a red is 10/30 or 1/3.

The probability of not picking a red chip is 20/30 or 2/3.

In this case we have 1/3 and 2/3.

Let's try one more, picking a king from the deck of cards.

What is the probability of picking a king? 4/52 or 1/13.

What is the probability of not picking a king? 48/52 or 12/13

Now we have 1/13 and 12/13.

What do you notice about each of these pairs of fractions? They add up to one.

If something has a probability of 0, how likely is it to happen (It can't happen). *For example, the probability of rolling a 7 on a 6-sided dice is 0. It can't happen.*

If something has a probability of 1, what is the likelihood that it will happen? You may have to write this probability as a fraction for the children to realize. *1/1 That means there is one favorable outcome and one possible outcome. That would be like rolling any number from 1-6 on six-sided dice. Your chances are certain.*

Probability Expressed as a Decimal and a Percent

Once children have learned how to convert standard fractions, decimal fractions, and percents, you need to show them that probability can be expressed in any of these forms.

For example: probability of 1/5 is the same as 0.2 or 20%.

Dependent and Independent Events

This lesson does not add a great deal of new information about probability. It only shows that if the probability of picking a red chip is 10/30 and I pick a white chip, if I do not return the chip to the bag, the probability has changed. The probability of picking a red chip now is 10/29.

These are called dependent events. Two events are dependent if the first outcome changes the probability of the second outcome.

If the children are struggling with this, you can contrast it with the same case replacing the chip each time. In that event, the probability of choosing a red chip will be 10/30 every time. The first outcome does not affect the second.

There are many other explorations that can be found in standard middle school math texts. Some examples of probability in real life include batting averages and weather forecasts.

Labels

Probability of an Event

Favorable Outcomes

Possible Outcomes

Trials

Independent Event

Dependent Event

Probability Expressed as a Fraction

Probability Expressed as a Decimal

Probability Expressed as a Percent